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Note

Effective Thermal Conductivity of a Matrix with Two Kinds of Inclusions

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Previous approximations for the effective conductivity of a matrix with inclusions of low conductivity, and for the effective resistivity of a matrix with inclusions of high conductivity, are combined to give the effective conductivity when both types of inclusions are present. This applies, for example, to a ceramic which contains both pores and metallic inclusions.

KEY WORDS: ceramics; colloids composites; conductivity; pores; resistivity; thermal conductivity.

1. INTRODUCTION

There have been several approximations for the effective electrical or thermal conductivities of inhomogeneous media, reviewed by Parrott and Stuckes [1] and by Allitt et al. [2] They have been applied mainly to two-component composites. Håkansson and Ross [3] measured the thermal conductivity of such composites and compared them to these various approximations. The present author has given an approximation which expresses the local conductivity as a spatial Fourier composition, yielding an expression in terms of the average conductivity and fluctuations about that average [4, 5]. A similar approximation can be obtained in terms of average resistivity and its fluctuations [6]. Like all perturbation theories, one must chose that version which minimizes the perturbation, in this case the fractional fluctuations of either the conductivity or the resistivity. Thus one should use the conductivity formulation for inclusions which have a resistivity much higher than the matrix; for inclusions which have a high conductivity one should use the resistivity formulation.

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For a volume fraction f_p of low-conductivity inclusions, of thermal conductivity λ_p, in a matrix of conductivity λ_m, and in the absence of any preferred orientation, the effective conductivity becomes [5]

$$\lambda_e = \lambda_m(1 - f_p) + \lambda_p f_p - \frac{f_p(1 - f_p)(\lambda_m - \lambda_p)^2}{3\lambda_p f_p + \lambda_m(1 - f_p)} \tag{1}$$

In the limit when $\lambda_p \ll \lambda_m$, as in the case of pores with negligible radiation, this becomes

$$\lambda_e = \lambda_m[1 - (4f_p/3)] \tag{2}$$

and this holds also for nonspherical pores, provided they are randomly oriented.

In the opposite case of highly conductive inclusions of thermal conductivity λ_i and volume fraction f_i, again in the absence of preferred orientations the resistivity approximation yields [6] an effective resistivity

$$W_e = W_m(1 - f_i) + W_i f_i - \frac{2}{3}\frac{f_i(1 - f_i)(W_m - W_i)^2}{W_m(1 - f_i) + W_i f_i} \tag{3}$$

where $W_m = 1/\lambda_m$ and $W_i = 1/\lambda_i$. Again, in the limit $W_i \ll W_m$, as for metallic inclusions in a ceramic matrix, this becomes

$$W_e = W_m[1 - (5f_i/3)] \tag{4}$$

Both approximations fail when the concentrations are in the middle range.

There are cases when there are two different inclusions in a matrix, one highly conductive ($\lambda_i \gg \lambda_m$, volume fraction f_i) and one of very low conductivity ($\lambda_p \ll \lambda_m$, volume fraction f_p). This may occur when a ceramic contains both pores and colloidal metal inclusions. Pores result from incomplete sintering, and additional pores or bubbles may form by the generation of helium during radioactive decay. Colloid inclusions or ordered second phases in a random solid solution may form either during initial formation of the solid or by subsequent radioactive decay or fission. To deal with the simultaneous presence of both types of inclusions, one can consider a matrix which has the effective conductivity of a ceramic with pores, using Eq. (1), and then use the corresponding resistivity to include the effect of colloids, using Eq. (3). Alternatively, one can use the inverse procedure of first calculating the resistivity of matrix and colloids, then including the effect of pores on the conductivity.

In the important limit of low volume fractions and when $\lambda_p \ll \lambda_m \ll \lambda_i$, both procedures lead to the same result:

$$\lambda_e = \lambda_m[1 - (4f_p/3)][1 - (5f_i/3)]^{-1} \tag{5}$$

In other cases, when the two procedures give slightly different results, one would first calculate an effective matrix conductivity with the more finely dispersed phase, using either Eq. (1) or Eq. (3), with a second step of including the other phase using either Eq. (3) or Eq. (1).

For small fractions, Eq. (5) becomes

$$\lambda_e \cong \lambda_m[1 - (4f_p/3) + (5f_i/3)] \tag{6}$$

This expression may suffice in many practical cases.

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